Model VI(a)-3

Yet another related model due to Lu (1989) has the following survival function:

$$\bar{F}(t_1,t_2) = \exp\left[-\lambda_1 t_1^{\beta_1} - \lambda_2 t_2^{\beta_2} - \lambda_0 \max(t_1,t_2)^{\beta_0}\right] \tag{2.57}$$

This can be seen as a slight modification (or generalisation) of the Marshall and Olkin's bivariate exponential distribution due to the exponent in the third term having a new parameter.

Model VI(a)-4

A general model proposed by Lu and Bhattacharyya (1990) has the following form:

$$\bar{F}(t_1,t_2) = \exp\left[-\left(\frac{t_1}{\alpha_1}\right)^{\beta_1} - \left(\frac{t_2}{\alpha_2}\right)^{\beta_2} - \delta\psi(t_1,t_2)\right] \tag{2.58}$$

Different forms for the function of $\psi(t_1,t_2)$ yield a family of models. One form for $\psi(t_1,t_2)$ is the following:

$$\psi(t_1,t_2) = \left[\left(\frac{t_1}{\alpha_1}\right)^{\beta_1/m} + \left(\frac{t_2}{\alpha_2}\right)^{\beta_2/m}\right]^m \tag{2.59}$$

This yields the following survival function for the model:

$$\bar{F}(t_1,t_2) = \exp\left\{-\left(\frac{t_1}{\alpha_1}\right)^{\beta_1} - \left(\frac{t_2}{\alpha_2}\right)^{\beta_2} - \delta\left[\left(\frac{t_1}{\alpha_1}\right)^{\beta_1/m} + \left(\frac{t_2}{\alpha_2}\right)^{\beta_2/m}\right]^m\right\} \tag{2.60}$$

Model VI(a)-5

This is another case of (2.58) with

$$\begin{aligned}\bar{F}(t_1,t_2) = \exp\Bigg(&-\left(\frac{t_1}{\alpha_1}\right)^{\beta_1} - \left(\frac{t_2}{\alpha_2}\right)^{\beta_2} - \delta\left\{1 - \exp\left[-\left(\frac{t_1}{\alpha_1}\right)^{\beta_1}\right]\right\} \\ &\times \left\{1 - \exp\left[-\left(\frac{t_2}{\alpha_2}\right)^{\beta_2}\right]\right\}\Bigg)\end{aligned} \tag{2.61}$$

Model VI(a)-6

Finally, the Morgenstern–Gumbel–Farlie system of distributions (Johnson and Kotz, 1970b) is given by

$$\bar{F}(t_1,t_2) = \bar{F}_1(t_1)\bar{F}_2(t_2)\{1 + \gamma[1 - \bar{F}_1(t_1)][1 - \bar{F}_2(t_2)]\} \tag{2.62}$$

With $\bar{F}_i(t_i) = \exp\{-t_i^{c_i}\}$, this yields a special case of the model given by (2.58).

The estimates are obtained by solving the following three equations simultaneously:

$$\hat{\lambda} = \frac{k}{\hat{\alpha}\sum_{i=1}^{k} \exp\left(t_{(i)}/\hat{\alpha}\right)^{\hat{\beta}} + (n-k)\hat{\alpha}\exp\left(t_{(k)}/\hat{\alpha}\right)^{\hat{\beta}} - n\hat{\alpha}} \tag{7.114}$$

$$\begin{aligned} &\frac{k}{\hat{\beta}} + \sum_{i=1}^{k} \ln\frac{t_{(i)}}{\hat{\alpha}} + \sum_{i=1}^{k}\left[\left(\frac{t_{(i)}}{\hat{\alpha}}\right)^{\hat{\beta}} \ln\frac{t_{(i)}}{\hat{\alpha}}\right] - \lambda\alpha\sum_{i=1}^{k}\left[\exp\left(\frac{t_{(i)}}{\hat{\alpha}}\right)^{\beta}\left(\frac{t_{(i)}}{\hat{\alpha}}\right)^{\hat{\beta}} \ln\left(\frac{t_{(i)}}{\hat{\alpha}}\right)\right] \\ &- (n-k)\hat{\lambda}\hat{\alpha}e^{\left(t_{(k)}/\alpha\right)^{\hat{\beta}}}\left(\frac{t_{(k)}}{\hat{\alpha}}\right)^{\hat{\beta}} \ln\left(\frac{t_{(k)}}{\hat{\alpha}}\right) = 0 \end{aligned} \tag{7.115}$$

and

$$\begin{aligned} -\frac{k(\hat{\beta}-1)}{\hat{\alpha}} + n\lambda - \frac{1}{\hat{\alpha}}\sum_{i=1}^{k}\left(\frac{t_{(i)}}{\hat{\alpha}}\right)^{\hat{\beta}} - \lambda\sum_{i=1}^{k}\left\{e^{\left(t_{(i)}/\hat{\alpha}\right)^{\hat{\beta}}}\left[1-\left(\frac{t_{(i)}}{\hat{\alpha}}\right)^{\hat{\beta}}\right]\right\} \\ - (n-k)\lambda e^{\left(t_{(k)}/\hat{\alpha}\right)^{\hat{\beta}}}\left[1-\left(\frac{t_{(k)}}{\hat{\alpha}}\right)^{\hat{\beta}}\right] = 0 \end{aligned} \tag{7.116}$$

7.13.4 Hypothesis Testing

Xie et al. (2002) discuss testing the hypothesis $\alpha = 1$ (Chen's model) versus $\alpha \neq 1$ based on the likelihood ratio test.

EXERCISES

Data Set 7.1 Complete Data: Failure Times of 20 Components

0.072	0.477	1.592	2.475	3.597
4.763	5.284	7.709	7.867	8.661
8.663	9.511	10.636	10.729	11.501
12.089	13.036	13.949	16.169	19.809

Data Set 7.2 Censored Data: 30 Items Tested with Test Stopped after 20th Failure[a]

0.0014	0.0623	1.3826	2.0130	2.5274
2.8221	3.1544	4.9835	5.5462	5.8196
5.8714	7.4710	7.5080	7.6667	8.6122
9.0442	9.1153	9.6477	10.1547	10.7582

[a] The data is the failure times.

Traffic Constraints

$$\lambda_{ij} \leq \lambda_{\max}, \quad \forall (i,j) \tag{4.4}$$

$$\lambda_{ij} = \sum_{sd} \lambda_{ij}^{(sd)}, \quad \forall (i,j) \tag{4.5}$$

$$\lambda_{ij}^{(sd)} \leq b_{ij}\lambda^{(sd)}, \quad \forall (i,j),(s,d) \tag{4.6}$$

$$\sum_j \lambda_{ij}^{(sd)} - \sum_j \lambda_{ji}^{(sd)} = \left\{ \begin{array}{ll} \lambda^{(sd)}, & s=i \\ -\lambda^{(sd)}, & d=i \\ 0, & s \neq i, d \neq i \end{array} \right\} \forall (s,d) \tag{4.7}$$

Wavelength Constraints

$$\sum_{k=0}^{W-1} c_{ij}^{(k)} = b_{ij}, \quad \forall (i,j) \tag{4.8}$$

$$c_{ij}^{(k)}(l,m) \leq c_{ij}^{(k)}, \quad \forall (i,j),(l,m),k \tag{4.9}$$

$$\sum_{ij} c_{ij}^{(k)}(l,m) \leq 1, \quad \forall (l,m),k \tag{4.10}$$

$$\begin{array}{l} \sum_{k=0}^{W-1} \sum_l c_{ij}^{(k)}(l,m) p_{lm} - \sum_{k=0}^{W-1} \sum_l c_{ij}^{(k)}(m,l) p_{ml} \\ = \left\{ \begin{array}{ll} b_{ij}, & m=j \\ -b_{ij}, & m=i \\ 0, & m \neq i, m \neq j \end{array} \right\} \forall (i,j),m \end{array} \tag{4.11}$$

Hop Constraints

$$\sum_{lm} c_{ij}^{(k)}(l,m) \leq h_{ij}, \quad \forall (i,j),k \tag{4.12}$$

Discussion Most of the above constraints are self-explanatory. Many of them enforce the consistency between the various parameters and variables of the formulation. Constraint (4.7) asserts the conservation of traffic at lightpath endpoints. Expression (4.11) asserts the conservation of every wavelength at every physical node for each lightpath.

The parameters, or inputs, to the formulation are the traffic matrix Λ, the hop bound matrix H, the number of wavelengths in a fiber W, the desired logical degree Δ_l, and the details of the physical topology graph. The variables, whose values at optimum are the “output” of the MILP, relate to the virtual topology graph, wavelength assignment in the virtual topology, and the traffic routing over the virtual topology. The lightpath indicators b_{ij} provide the